J. C. Chedjou
International Centre for Theoretical Physics **(ICTP)**, Strada Costiera 11 34014 Trieste, Italy and Department of Physics, Faculty of Science, University of Dschang, BP 67, Dschang, Cameroon.
chedjou@ictp.it or chedjou@moniut.univ-bpclermont.fr

L. K. Kana
University of Dschang, **(UDETI)** BP 67, Dschang, Cameroon.
klluca@yahoo.fr

I. Moussa
Department of Physics, Faculty of Science, University of Yaoundé-I, BP 812, Yaoundé, Cameroon.
moussaildoko@yahoo.fr

K. Kyamakya
University of Klagenfurt, Chair of Computer Science in Transportation, Univ str. 65 A9020, Klagenfurt, Austria
kyamakya@isys.uni-klu.ac.at

A. Laurent
Blaise Pascal University, LEM-IUT, GEII, Avenue Aristide Briand, 03101, Montluçon Cedex, France
laurent@moniut.univ-bpclermont.fr

Dynamics of a Quasi-periodically Forced Rayleigh Oscillator

This paper studies the dynamics of a self-excited oscillator with two external periodic forces. Both the nonresonant and resonant states of the oscillator are considered. The hysteresis boundaries are derived in terms of the system's parameters. The stability conditions of periodic oscillations are derived. Routes to chaos are investigated both from direct numerical simulation and from analog simulation of the model describing the forced oscillator. One of the most important contributions of this work is to provide a set of reliable analytical expressions (formulas) describing the system's behavior. These are of great importance to design engineers. The reliability of the analytical formulas is demonstrated by a very good agreement with the results obtained by both the numeric and the experimental analysis.

Keywords - Oscillatory states, Hysteresis boundaries, Stability criteria, Nonlinear oscillator, Chaos, Bifurcations, Analog Simulation.

1 Introduction

In recent years a twofold interest has attracted theoretical, numerical and experimental investigations to understand the behavior of nonlinear oscillators. The theoretical (fundamental) investigation reveals their rich and complex behavior and the experimental (self-excited oscillators) describes the evolution of many biological, chemical, physical, mechanical and industrial systems [1-3]. Recently the chaotic behavior of these oscillators is exploited in the field of communication for coding information [4]. The periodically forced Rayleigh equation has been extensively studied in the investigation of the response of a self-excited oscillator when it is driven by a periodic force [2]. It is the first model of irregular oscillations (transient chaos or ''ghost solutions'') of differential equations [5, 6]. To the best of our knowledge, little has been done in a system consisting of a self-excited Rayleigh oscillator with two external periodic forces. The choice of an external force with two periodic components gives the possibility of different types of excitations (sinusoidal, quasi-periodic, time domain amplitude modulated, relaxative and so on). This paper aims at the following: a) considers the dynamics of the system; b) contributes to the general understanding of the behavior of the system and points out some of its unknown behavior; c) experimental investigation of the dynamics of the system.

The dynamics of the forced Rayleigh oscillator is described by the following nonlinear differential equation:

$$\ddot{x} - \varepsilon_1\left(1-\dot{x}^2\right)\dot{x} + \omega^2 x = f(t), \qquad (1a)$$

where ε_1 and ω are positive parameters, being respectively the damping coefficient and the natural angular frequency.

Equation (1a) when $f(t)=0$ exibits a sinusoidal behavior for small ε_1 and leads to a relaxation oscillation for large ε_1 . The latter is suited for the control in systems with input stimulus (that produces a response of fixed amplitude) but adaptable frequency or repetition rate. This is similar to a beating heart when each contraction of the ventricule is stimulated by a nerve impulse generated upon contraction of the auricle [3]. A self-excited Rayleigh oscillator can also be used in communication in its autonomous mode for small ε_1 , as a sinusoidal oscillator amplitude control, the natural frequency ω being the control parameter. A sinusoidal function generator is an important circuit which is used in measurements, instrumentation, telecommunications and electronics, just to name a few.

In the presence of an external excitation interesting phenomena (phase locking, collapse of torus and folded-torus) are observed [6].

In this paper we concentrate on the analysis of equation (1a) when

$$f(t) = k_1 \cos\left(\omega_1 t + \theta_1\right) + k_2 \cos\left(\omega_2 t + \theta_2\right). \qquad (1b)$$

[7] considers the analytical approach to torus bifurcations and [8] the analytical and numerical study of the Duffing oscillator subjected to two external periodic forces.

The paper is structured as follows. Section 2 gives an analytical treatment of equations (1). Approximate solutions of equations (1) in the nonresonant case are obtained with the