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Behavior of a Self-Sustained Electromechanical Transducer and Routes to Chaos

This paper studies the dynamics of a self-sustained electromechanical transducer. The stability of fixed points in the linear response is examined. Their local bifurcations are investigated and different types of bifurcation likely to occur are found. Conditions for the occurrence of Hopf bifurcations are derived. Harmonic oscillatory solutions are obtained in both non-resonant and resonant cases. Their stability is analyzed in the resonant case. Various bifurcation diagrams associated to the largest one- dimensional (1-D) numerical Lyapunov exponent are obtained, and it is found that chaos can appear suddenly, through period- doubling, period- adding or through torus breakdown. The extreme sensitivity of the electromechanical system to both initial conditions and tiny variations of the coupling coefficients is also outlined. The experimental study of the electromechanical system is carried out. An appropriate electronic circuit (analog simulator) is proposed for the investigation of the dynamical behavior of the electromechanical system. Correspondences are established between the coefficients of the electromechanical system model and the components of the electronic circuit. Harmonic oscillatory solutions and phase portraits are obtained experimentally. One of the most important contributions of this work is to provide a set of reliable analytical expressions (formulas) describing the electromechanical system behavior. These formulas are of great importance for design engineers as they can be used to predict the states of the electromechanical systems and respectively to avoid their destruction. The reliability of the analytical formulas is demonstrated by the very good agreement with the results obtained by both the numeric and the experimental analysis.

Keywords- Electromechanical Transducer, Oscillatory states, Analog simulation, Coupled non-linear oscillators, Bifurcations, Chaos.

1 Introduction

In recent years, there has been a huge interest in the dynamics of coupled nonlinear oscillators or modes [1 – 10]. This is due to the fact that coupled nonlinear oscillators or modes can describe the dynamics of various physical, electromechanical, chemical and biological systems. Among these coupled nonlinear oscillators, the most intensively studied are two coupled Duffing oscillators with quadratic and cubic nonlinearities [1, 6], two coupled van der Pol oscillators [2 – 5] and the coupling between the van der Pol and Duffing type oscillators [7, 9, 10, 11, 12, 13]. Besides, a model of coupled attractors of different types (coupling between the van der Pol oscillator (the limit cycle prototype) and a Duffing oscillator (the strange attractor prototype)) might serve as a good model for real systems in nature [13]. Such a coupled system may represent the kind of self-sustained oscillating system that shows some phenomena such as hysteresis, quenching, and, to name a few others, resonant and anti-resonant phenomena that may be found in a physical, a biological, an economic or an electromechanical system [13].

Concerning systems consisting of a van der Pol oscillator coupled to a Duffing oscillator, some interesting works have been carried out. Ref. [11] does explain the competition between both a van der Pol oscillator and the Duffing oscillator. The elastic coupling between the two types of oscillators is considered and, it is shown that the destabilization of the two traditionally stable steady states does lead to the birth of quasi-periodic orbits. It is also shown that the increase of the coupling strength results in the destruction of the limit cycle and the birth of a chaotic dynamics, while its decrease leads to the stabilization of the limit cycle. Ref. [12] deals with the analysis of numerical solutions for a system of two van der Pol-Duffing oscillators with nonlinear coupling. The existence of chaotic switching between two oscillatory regimes differing by nearly time-constant phase shifts between the coupled subsystems is shown. Bifurcations of the periodic motions corresponding to synchronization of two subsystems are investigated. Both stability regions of synchronization regimes and routes to chaos are found. The authors of Ref. [13] considered the dynamical behavior of a system consisting of a van der Pol oscillator coupled to a Duffing oscillator. They showed that the existence of different attractors in a system does generate very rich dynamic phases when monitoring both the system parameters and the coupling constant. They further examined the transition between dynamic phases by constructing the bifurcation diagram and the phase diagram of the parameter space for a specific coupling constant. They found that the dynamics of the system becomes simpler for large values of the coupling constant. They showed the achievement of the synchronization for large values of the coupling constant. In Ref. [7] we studied the dynamics of a system consisting of a van der Pol oscillator coupled to a Duffing oscillator. The couplings are dissipative (coupling through velocities) and