Harmonic Oscillations, Routes to chaos and Synchronization In a Nonlinear Emitter-Receiver System

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Abstract

This paper considers a model describing the dynamics of a Transmitter-receiver system in communication engineering. The bifurcation structure is analyzed to define the nature of the bifurcations exhibited by the system. One does found both the complex dynamical behavior of the system and its extreme sensitivity to tiny changes in the model parameters. Crisis, Period-doubling and Hopf bifurcation are also found. An analog simulation of the Transmitter-receiver system is carried out. Some experimental phase portraits are obtained. Both regular and chaotic modulations of the incoming message are done experimentally. A comparison of the results obtained from experimental and numerical analysis shows a very good agreement.

I. INTRODUCTION

Recently, there has been much interest in the wish of applying nonlinear phenomena and chaos in physics and engineering science [1, 2]. The interest devoted to these phenomena is due to their various applications: Chaotic modulation [3, 4], synchronization and secure communication [5 - 8] just to mention few applications. In the latter application, tremendous interest is observed since the pioneering work of Pecora and Carroll [9]. This potentially rich and fertile field based on chaotic synchronization has recently gained much interest, mainly in the wish of applying it in physics and engineering science. Concerning synchronization and secure communication, the problem to deal with consists in processing the received signal in order to construct the message injected into the chaotic model at the transmitter.

This paper considers a transmitter-receiver system described by the following model:

$$y + ay + y + by^3 - f\left(x + \frac{x^2}{2}\right) = E\cos\omega t \qquad (1a)$$

$$x + cx + d(x - y - xy) = \mathrm{G}\cos\gamma t \qquad (1b)$$

where y and x are the coordinates of both oscillators, b the nonlinear coefficient, a and c the dissipative parameters, f and d are the coupling coefficients. E and G are the amplitudes of the messages. Our study was stimulated by earlier works on the Emitter-receiver system [10, 11]. In ref. [10] we used the multiple time scales method to derive harmonic oscillatory solutions both in the resonant and nonresonant cases. Melnikov theorem was used to obtain basins of attraction of chaotic solutions. Routes to chaos were analyzed numerically. Ref. [11] is mainly focused on the numerical analysis of the system. A real physical prototype whose dynamics is described by Eqs. (1) is presented. The description of the prototype is carried out. Transitions to chaos are analyzed: period- doubling and crisis are observed. Our aim in this paper is:

- To consider the dynamics of such a system (see ref. [11] for the description of the physical model);
- To contribute to the general understanding of the behavior of this system and complete the results obtained so far by pointing out some of its unknown behavior.
- To carry out the experimental study of the system.

II. STABILITY AND BIFURCATION STRUCTURES

Eqs. (1) can be transform into a non autonomous system of the first order differential equation of the form

$$x = v \qquad (2a)$$

$$v = -cv + d\left(y + xy - x\right) \qquad (2b)$$

$$y = w \qquad (2c)$$

$$w = -aw - y - by^3 + f\left(x + \frac{x^2}{2}\right) + E\cos\omega t\,. \qquad (2d)$$

By perturbing Eqs. (2) around the steady state $\left(x_0, v_0, y_0, w_0\right)$ we obtain the following 4X4 jacobian matrix

$$M_J = \begin{bmatrix} 0 & 1 & 0 & 1 \\ d\left(y_0 - 1\right) & -c & d\left(1 + x_0\right) & 0 \\ 0 & 0 & 0 & 1 \\ f\left(1 + x_0\right) & 0 & -\left(1 + 3by_0^2\right) & -a \end{bmatrix}. \qquad (3)$$

The stability of the periodic motion is determined according to the real parts of the roots of the following characteristic equation $\left(\det\left(M_J - \lambda I_d\right) = 0\right)$:

$$\eta^4 + (c + a)\eta^3 + (1 + ac + d)\eta^2 + (c + ad)\eta + d(1 - f) = 0 \qquad (4)$$

obtained by considering very small amplitudes of the steady states.

Fig. 1 shows in the complex plane $\left(\mathrm{Re}(\eta), \mathrm{Im}(\eta)\right)$ the representation of the eigenvalues (roots of the characteristic equation). These roots are obtained (using the Newton-Raphson algorithm) for $0.25 \le a \le 1.75$, $-1 \le f \le 1$, $c = 0.05$ and $d = 0.5$. From Fig. 1 one can have the idea on both the stability of periodic solutions and on different types of bifurcation likely to appear in the system. M_J, being a real matrix, complex eigenvalues occur in complex conjugate pairs responsible for the observed symmetric along the real axis. Thus if the real parts of the eigenvalues (η) are all negative, the rate is of the contraction type or otherwise of the expansion [12]. If the eigenvalues are all real, the contraction or expansion are observed near the steady state while the complex values of the eigenvalues show the contraction or expansion of the spiral [12].

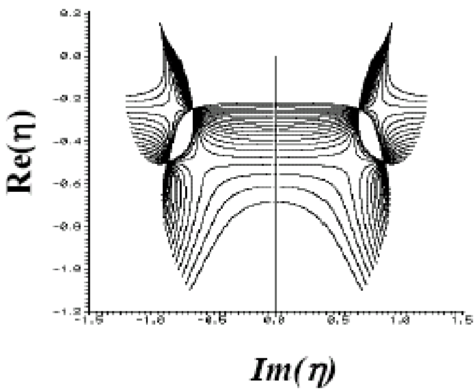

Figure.1 Representaion of the eigen values solutions in the complex frame

If there exists eigenvalues having real parts with different sign, the equilibrium state is called saddle; an equilibrium point whose eigenvalues all have a non zero real parts are called hyperbolic [13]. On the other hand period-doubling bifurcation is observed if there exists an eigenvalue $\eta = -1$ while bifurcations of the Hopf type are observed if the following conditions are satisfied : a) there exists a pair of pure imaginary complex conjugate eigenvalues. b) $\left.\frac{d\eta}{d\alpha}\right|_{\alpha=\alpha_c} \neq 0, \alpha$, being the bifurcation control parameter. α_c is the critical value for the occurrence of Hopf bifurcation, obtained from the equation $\mathrm{Re}(\eta) = 0$ [12]. We have derived the following critical relationships between the parameters of the model (describing the dynamics of the system) for the occurrence of period-doubling bifurcation

$$f = \frac{a(c-d-1) + 2(1+d-c)}{d} \qquad (5)$$

and Hopf bifurcation

$$f = \frac{\left(-ac\left((d-1)^2 + ac(1+d) + a^2 d + c^2\right)\right)}{d(a+c)^2} \qquad (6)$$

Considering the previous results it clearly appears (see Fig. 1) that our system of the given parameters a and f can undergo various types of bifurcations namely: saddle, period-doubling, Hopf bifurcation, and a symmetry-breaking bifurcation which is often a prerequisite for the first period- doubling bifurcation [14]. Kozlowski et al. [15] considered a coupled identical single-well Duffing oscillators and showed by linear analysis a similar eigenvalues scenario. Except Hopf bifurcation, such bifurcations have been successfully found in one-dimensional (1-D) Double-well Duffing oscillators subjected to a periodically driven force [16 - 19]. The above analysis serves both to predict the local instability of the steady state and to be aware of the type of bifurcation expected in the system.

III. EXPERIMENTAL STUDY

We propose an electronic simulator (see Fig. 2) for the experimental investigation of the dynamics of our system. Using an appropriate time scaling, the simulator's outputs can be viewed directly on an oscilloscope by simply feeding the voltages to the X-input and Y-input of the oscilloscope.

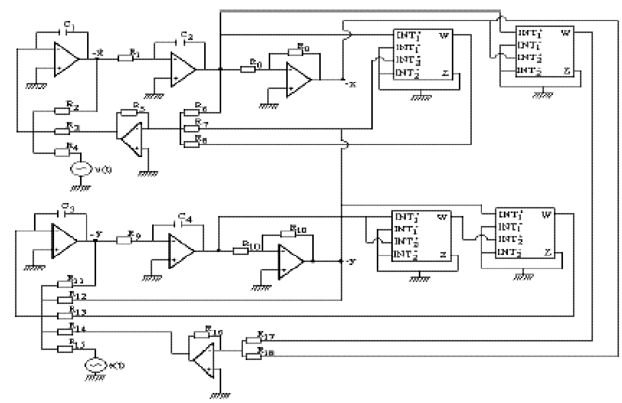

Figure 2. Theme of the electronic simulator

Considering the electronic simulator (Fig. 2) it can be shown that the voltages at point x and (outputs of opams) are described by the set of coupled model (Eqs. 1). y
In terms of the circuit components, the parameters of Eqs. (1) are defined as follows :

$$a = \frac{1}{10^4 R_{11} C_3}; \; b = \frac{1}{10^{10} R_9 R_{13} C_3 C_4};$$

$$c = \frac{1}{10^4 R_2 C_1}; \; d = \frac{R_5}{10^8 R_1 R_3 R_6 C_1 C_2};$$

$$E = \frac{A_2}{10^8 R_9 R_{15} C_3 C_4}; \; f = \frac{R_{16}}{10^8 R_9 R_{14} R_{18} C_3 C_4};$$

$$G = \frac{A_1}{10^8 R_1 R_4 C_1 C_2};$$

taking into account the following critical relationships: $R_6 = R_7 = 10 R_8$ and $R_{18} = 5 R_{17}$; A_1 and A_2 are the amplitudes of the generators.
In order to control each parameter of Eqs. (1) by varying only one resistor, we set the following values of the components : $R_0 = R_1 = R_5 = R_6 = R_7 = R_9 = R_{10} = R_{16} = R_{18} = 10000\Omega$; $C_1 = C_2 = 10.73nF$; $C_3 = C_4 = 10.16nF$; Thus, the coefficients $a, b, c, d.E, f$ and G will respectively be controlled by $R_{11}, R_{13}, R_2, R_3, R_{15}, R_{14}$, and R_4. Note that the analog voltages obtained from our simulator are directly equivalent to the dimensionless variables of Eqs. (1).

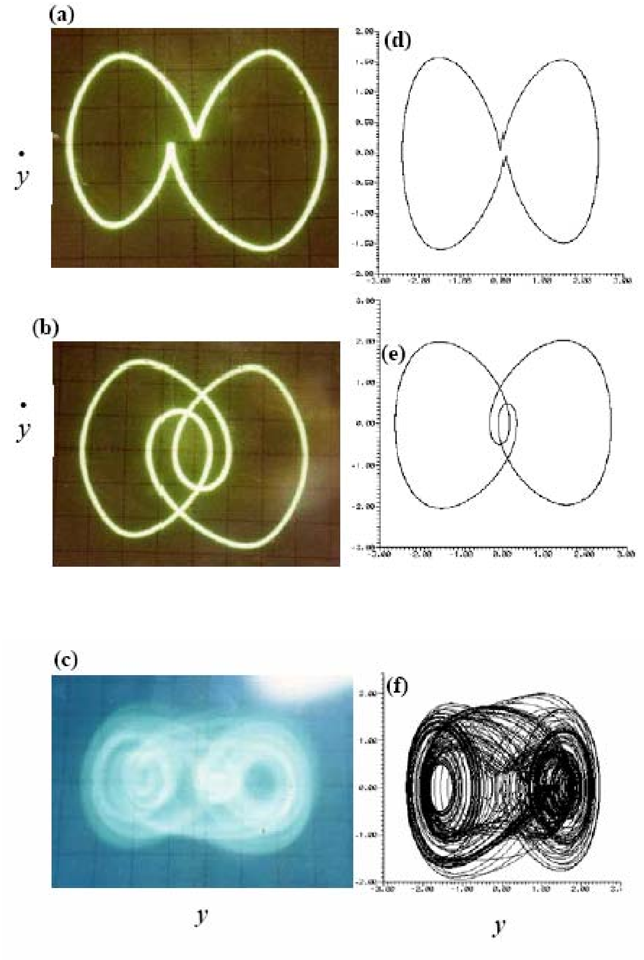

Figure 3: Phase portraits: Experimental values:
(a) $\omega = 0.540 rad/s$; (b) $\omega = 0.496 rad/s$;
(c) $\omega = 0.0302 rad/s$; Numerical values:
(d) $\omega = 0.534$; (e) $\omega = 0.497$; (f) $\omega = 0.306$

A. Bifurcation and onset of chaos

This sub-section is devoted to the experimental findings of the various bifurcations and types of motions that can occur in our system when one component of the analog circuit is monitored. Our control component is the frequency of the excitation ω .

We consider the case where the excitation has no effect on the whole system $(R_4 = \infty)$ with the following values of resistors:

$R_2 = 1800\Omega$; $R_3 = 3780\Omega$; $R_8 = 1000\Omega$: $R_{11} = 46000\Omega$; $R_{12} = 9690\Omega$; $R_{13} = 421\Omega$: $R_{14} = 866000\Omega$; $R_{15} = 5000\Omega$; $R_{17} = 2000\Omega$; $A_2 = 1.3V$

Thus, the corresponding parameters take the following values:
$a = 0.022068$; $b = 0.23$; $c = 5.177$; $d = 2.29778$
$E = 2.5345$; $f = 0.01118$.

We have found, using this set of parameters that as ω is monitored, the oscillations follows a period-adding routes to chaos: we have observed (Petriod-1 ⟶ Period-3 Peiod-5 ⟶ Period 7 ⟶ Quasiperiodic Chaos) scenario routes to chaos. The pictures (a), (b) and (c) in Fig. 3 show some experimental phase portraits obtained within the period- adding sequence. Period-1(a), period- 2 (b), chaos (c) attractors are shown.

In order to confirm the results from our electronic simulator, we have carried out a direct numerical simulation of Eqs. (1). The phase portraits (d), (e) and (f) of Fig. 3 are respectively the corresponding numerical phase portraits of (a), (b) and (c) obtained using the same sets of system parameters. The numerical results confirm the sequence obtained experimentally. Let us mention that the sequence of bifurcations obtained when monitoring the frequency of the excitation are identical to those obtained by controlling the amplitude of the excitation. We have found that the experimental phase portraits are very similar to the numerical ones. We have considered the following set of system system parameters:

$a = 0.022068$; $b = 0.23$; $c = 5.177$; $d = 2.29778$; $E = 2.5345$; $f = 0.01118$

To analyze the nature of transitions to chaos numerically Figs. 4 show a bifurcation diagram of the attractor x associated to the graph of the largest one dimensional (1-D) Lyapunov exponent when the control parameter E is monitored. It shows the complex dynamical behavior exhibited by the system. As E increases the system follows the following complex bifurcation: torus destruction route to chaos, period- 8 crisis route to chaos and period-doubling route to chaos. Windows of regular motion alternate with windows of chaotic motion. The very weak ''chaoticity'' of the system is shown.

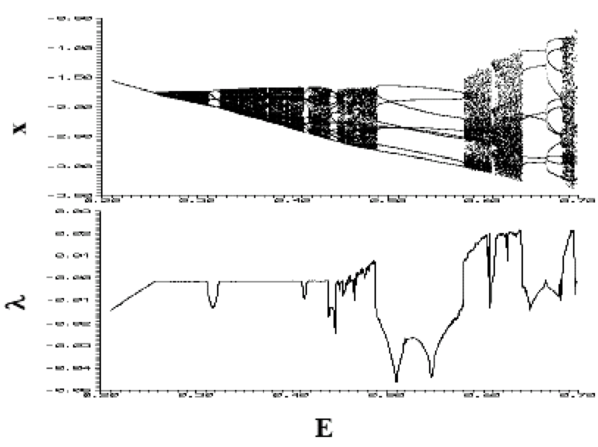

Figure 4: Bifurcation diagram of the attractor x associated to the largest 1-D Lyapunov exponent.

B. Modulation

Our aim in this sub-section is to show that for a given set of system parameters, the incoming message can be modulated. We set the following values of the circuit components:

$R_0 = R_{10} = 26800\Omega$; $R_2 = 2600\Omega$; $R_3 = 6840\Omega$;
$R_9 = 2110\Omega$; $R_{11} = 820000\Omega$; $R_{12} = 23900\Omega$;
$R_{13} = 9100\Omega$; $R_{14} = 306000\Omega$; $R_{15} = 8300\Omega$;
$R_{16} = 2200\Omega$; $R_{17} = 2000\Omega$; $A_2 = 1.174V$;
$\omega = 1.2560 rad/s$.

The pictures of Fig. 5 show the regular modulation of the received signal x (Fig. 5a) and its chaotic modulation (Fig. 5b) when the frequency ω of the excitation is varied. We have observed during our experiment that the modulation phenomenon is very sensitive to tiny changes in ω. The regular modulation shows the 'Zener effect' of the carrier while the chaotic state shows its complete destruction.

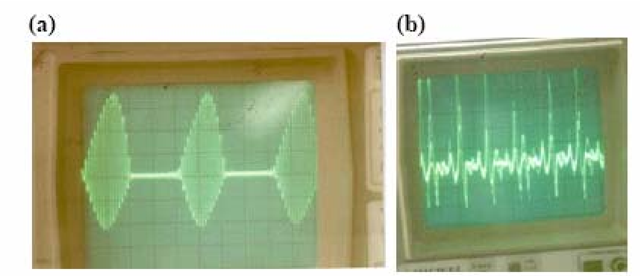

Figure. 5: Regular modulation (5. a) and chaotic modulation (5. b) of the received signal.

IV. CONCLUSION

This paper has presented the study of the dynamical behavior of an Emitter-receiver system. The bifurcations structure was analyzed and its complexity was proved through the plot of bifurcation diagrams. Analytic conditions for the appearance of period- doubling and Hopf bifurcation were derived. The Emitter-receiver system was studied experimentally. An appropriate electronic circuit was proposed for the experimental study of the system. Using both Kirchhoff current law and Newton dynamical law, the coefficients of the system model were derived in terms of the circuit components. Various experimental phase portraits were obtained showing the nature of transitions to chaos. Period-adding sequence was observed experimentally. We found period 8 sudden transition route to chaos, period- doubling route to chaos, and Hopf bifurcation. The regular and chaotic modulation of the received signal were also found.

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