On the evaluation of analog simulation of the dynamics of nonlinear systems for communication

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Abstract

This paper is focused on the evaluation of analog simulation of nonlinear systems for communication. Some problems encountered when performing analog simulations are listed. We propose some techniques to tackle them. Our motivation is to encourage scientists to use analog simulation technique for the analysis of systems despite the enormous focus on the numerical technology that kept analog advances a bit in the dark during the past decades. We discuss the advantages of the analog computation technology by presenting it versus the digital computation. To illustrate the concepts we choose a system model describing the dynamics of a Rayleigh oscillator submitted to an external quasi-periodic excitation. Sample results are presented using analog simulation. The results are compared with those from numerical simulation and a very good agreement is obtained. The main interest of this work is to prove that analog simulator is suitable than its numerical counterpart for the analysis of nonlinear physical problems.

I. INTRODUCTION

The electronic analog computer is basically a set of building blocks, each able to perform specific mathematical operations on direct current voltages and capable of being easily interconnected one to another. Some of the basic operations include addition, subtraction, multiplication, division, inversion, integration, and differentiation. By interconnecting these building blocks, mathematical equations are modelled. This computer establishes definite prescribed relations between continuously variable physical quantities.

The enormous focus on numerical technology during the past decades kept analog advances a bit in the dark. But recently, there has been a huge interest in using analog computer implementation technology to analyze nonlinear physical problems [1-6]. This is motivated by the fact that analog implementation offers the way to tackle the following difficulties encountered when using the most common numerical approach to solve nonlinear physical problems: Integration discontinuities, slow time integration (slow integration speed), and undefined duration of transient phenomena, to name a few.

The analog computer today finds its greatest application in the investigation of the dynamic behaviour of physical systems [1-6]. This computer has the following merits: artificially reproducing the phenomena, described in the mathematical formula to be computed, in the form of physical quantity, running operation to obtain the results, which are recovered to mathematical value representing the solution.

The analog simulation is of very great interest in Ultra Wide-Band (UWB) applications since it offers the possibility to simulate the behavior of systems at very high frequencies by performing an appropriate time scaling. By scaling time as an independent variable, physical processes that happen quickly (high frequency phenomena) can be stretched out. This serves to underscore the advantages of the scaling process, since it is very difficult to quantify very high frequency phenomena experimentally.

We discuss in this report the advantages of the analog computation technology in the analysis of nonlinear physical systems. Both analog and numerical simulation techniques are briefly described and compared. Invariably the question arises - Which is better, analog or numerical computation? Our wish is to show that both techniques are complementary, since each of them can analyze cases too complex for the other.

The general goal of this work is to present and prescribe some practical advises when dealing with analog implementation techniques. Our motivation is due to the fact that in the engineering field, this technique is not commonly used to solve nonlinear problems because of the saturation and offset phenomena of discrete components (diodes, transistors, operational amplifiers, and multipliers) of the electronic circuit (analog computer) built, the dynamics of the circuit being limited by the polarizing power supply (or static bias) of the discrete components. Moreover, the accuracy of an analog computer is limited by the accuracy of electrical components. Hopefully, by proposing some techniques to tackle the problems encountered during the implementation of analog computers it will encourage engineers to use analog implementation techniques for the analysis of nonlinear problems.

II. ANALOG COMPUTER Vs DIGITAL COMPUTER

The most common approaches to the problem of investigating the dynamics of nonlinear systems are the numerical simulation associated to the well-known analytical perturbation methods. However, it is well known that with the numerical technique problems related to time integration appear. In fact, even with very fast workstations, scanning parameter spaces turns out to be a very slow process [12]. Moreover, to the best of our knowledge, there exists no method that can help to predict the duration of the transient phase of a numerical simulation. Though the analog implementation is always limited by the saturation and offset phenomena of analogue devices such as operational amplifiers (LM741 and LF351) and multipliers (AD-633JN), it offers the way to tackle the following difficulties: integration discontinuities, slow time integration, transient phenomena duration undefined, to name a few. These are some major reasons for the increasing interest devoted to this type of simulation for the analysis of nonlinear and chaotic physical systems [12 – 19]. In fact, a properly designed circuit can provide sufficiently good real-time results faster than a numerical simulation on a fast computer [12]. Such a circuit must use high precision resistors and capacitors. In addition, the offset voltage of the operational amplifiers and multipliers must be well controlled.

Digital computation has been, since the thirties, the most important computational model, mainly due to the unifying work of Turing. Turing clarified the notion of algorithm, giving it a precise meaning, and introduced a coherent framework for discrete computation. With the rapidly growing needs of various fields such as physics and engineering, to make enormous quantities of calculations and information processing, many times beyond human capabilities, new computing devices were developed and improved. With these new technologies, digital computers improved dramatically in speed, size and accuracy, until the present date. This clarifies why digital (discrete) computation became today's main computational paradigm. In digital computation the internal states are discrete. Here each real is represented (and approximated) by strings of digits, whereas in analog computation, each real is handled exactly and is considered to be an intrinsic quantity.

The real philosophy of analog simulation doesn't take into account a notion of "algorithm" and there is no need to translate quantities into appropriate symbolic forms. In an analog simulator, variables are represented by physical quantities on which the operations are performed. The simulation is carried out by some physical systems that obey the same mathematical relations that control the physical or technical phenomenon under investigation [7]. This procedure is in some sense more natural to the physicists and to the engineers [8, 9]. A virtue of the analog simulator is that its basic design concepts are usually easy to recognize. What goes on inside is understandable since it is an analog of the real thing whereas the numerical types simulator is a product of pure logic. It cannot be described as similar to something with which we are familiar [10, 11].

Therefore, although digital computers had long ago superseded the analog simulators due, to a large extent, to the spectacular development of digital technology, we still believe that analog simulators might bring some fresh air to the theory of computation in the field of non linear dynamics.

III. PRACTICAL PROBLEMS AND ADVICES

Various practical problems are currently encountered during the implementation of analog simulators. Among these problems are some that automatically induce errors in analog calculations. This subsection aim both to list some practical problems encountered and proposes some practical advices to overcome them

A. Offset phenomenon

This phenomenon is the presence of a static voltage at inputs of analog devices (operational amplifiers (opamps), circuits multipliers, …) when they are supplied and sustained by a direct current polarizing power supply.

Fig. 1 shows a technique to cancel the offset phenomenon of an operational amplifier. By monitoring the potentiometer (P), it is easy to measure the evolution of the dc voltage at inputs of the opamps. One should avoid the situation P = 0 Ω that leads automatically to the destruction of the device. The potentiometer is monitored to transform the magnitude of the input voltages of opamps into almost the same order.

The offset cancellation becomes very complex when the electronic circuit is of a self-sustained type. In this case, the voltages at inputs of the analog device can be a direct consequence of the self-sustained character of the circuit. When the value of the self-sustained voltage at inputs of analog devices can be predicted, the method in Fig. 1 is used to fixe the predicted values.

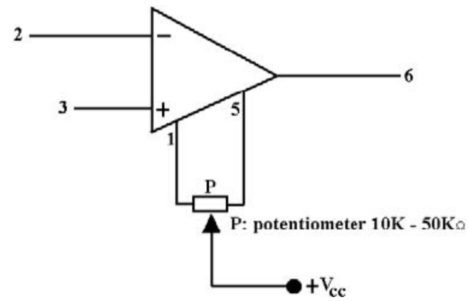


Figure 1: Illustration of the technique we use to cancel offset phenomena of operational amplifiers.

B. Saturation phenomenon

The dynamics of an analog simulator is limited by the power supply of analog devices. Saturation occurs when a voltage greater than that of the polarizing power supply can be found at a given point in the electronic circuit.

To overcome the saturation problem, the scaling factor notion is applied. We use a "**Static Check**" to verify if the system has been wired correctly. By tracing through the system we can calculate what the output voltage of each component should be. If it is determined that all outputs are of correct magnitude and sign (when measuring them), it can be safely assumed that the system is wired correctly.

C. Power transfer

When the electric current is flowing from a master device (transmitter) to a slave (receiver), the power is transferred in the same direction. A situation may arise where the power is not transferred. This can be explained by the fact that the dynamical resistor at the output of the master is not of the order of that at the input of the slave. Such a situation can be overcome easily by adapting the total dynamical resistor between the two devices. This is generally achieved by adding, in parallel, a dynamical resistor at the output of the first device (master) or at the input of the second (slave). The power may also not be transferred because the connection between the master and the slave is open. This problem can be detected by measuring the voltage at each point of the analog circuit.

D. Defective components

Analog components defect is generally caused by their wrong supply or by a complete shunt between their inputs. In their defective state, their temperature is very high. Many modules for a direct test of defective components are available. Concerning opamps, they can be tested as voltage device followers.

IV. IMPORTANT CONTRIBUTIONS

Let us mention the fact that some important proposals are available to tackle problems encountered by numerical simulation.

To tackle the numerical problem due to the integration discontinuities related to the choice of the step size, Thomas Rübner-Petersen [20] proposed an efficient algorithm using backward time-scale differences for solving stiff differential-algebraic systems. The proposed approach has computational advantages in simplicity and flexibility with respect to variations of the integration order k. In fact, this algorithm allows the order within each step to be changed in an optimal way between 1 and $k+1$. The implementation of the algorithm is described as part of a nonlinear analysis program, which has proved to be quite efficient for simulations of electronic networks. This program provides parameters in the DC analysis mode to be varied with automatic control of the step size. We have found that though the proposed method is very interesting in solving the numerical convergence problem since it varies automatically the step size to obtain an appropriate converging one, it requires very long time integration. Moreover, the integration duration becomes much more large because it increases with increasing nonlinearity in the system under investigation.

The community providing usable technical solutions for *Windows*^{tm} based PCB design **(or/CAD)** has proposed the possibility of using **GEAR** algorithm [21] in **Spice** to overcome the divergence problem due to an inappropriate choice of the integration step size [22]. The proposed method, though quite interesting, is limited by the fact that the simulation using **Spice** is still a theoretical analysis because the characteristics (or the internal parameters) of the analog components (Diodes, Transistors, Operational Amplifiers, and Multipliers) are chosen to be ideal (that is are not real).

Mention that Pspice and Matlab are currently used calculation tools for analog analysis rather than a real physical implementation (see the subsection below). These simulation tools are purely theoretical because the analog components they use are generally considered in the states where their characters are ideal. Hence it is still convincing that analog simulator is more suitable than the others for the analysis of nonlinear phenomena. It is a best tool to detect some strange phenomena such as chaos, modulation, demodulation and also synchronization to name a few.

V. SAMPLE RESULTS TO ILLUSTRATE THE CONCEPT

We consider the following nonlinear model:

$$\frac{d^2x}{dt^2} - \varepsilon_{01}\left(1-\left(\frac{dx}{dt}\right)^2\right)\frac{dx}{dt} + \omega^2 x = k_{01}\cos(\omega_1 t+\theta_1)+k_{02}\cos(\omega_2 t+\theta_2) \quad (1)$$

describing the dynamics of a Rayleigh oscillator subjected to a quasi-periodic external excitation. ε_{01} is the damping coefficient and ω the natural angular frequency, both being positive parameters. k_{01} and k_{02} are the amplitudes, ω_1 and ω_2 the angular frequencies, θ_1 and θ_2 the initial phases of the excitation

Figs. 2a and 2b are respectively the scheme of an appropriate analog simulator for the analysis of the dynamics of a forced Rayleigh oscillator and its real physical implementation.

In terms of the circuit components, the parameters of equation (1) are defined as follows:

$$\omega = \frac{10^{-4}}{\sqrt{R_1 R_4 C_1 C_2}}\ ; \qquad \varepsilon_{01} = \frac{10^{-4} R_3}{R_2 R_5 C_1}\ ;$$

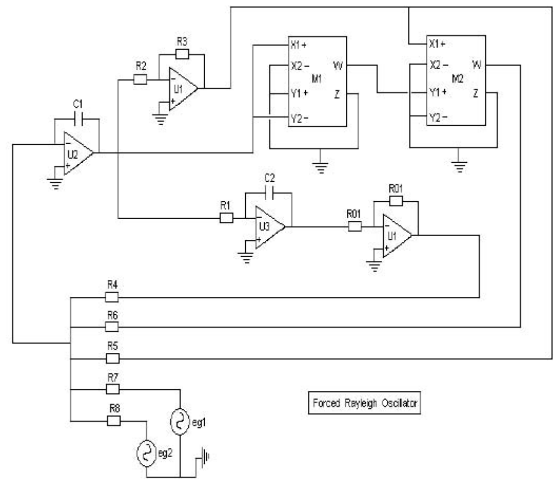


Figure 2a. Schematic of the electronic simulator.

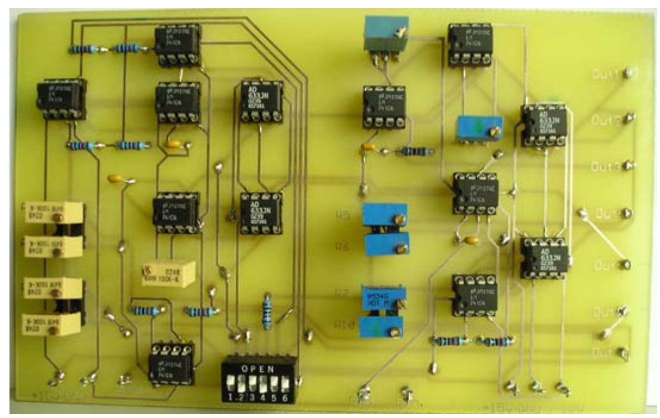


Figure 2b. Implementation of the electronic simulator.

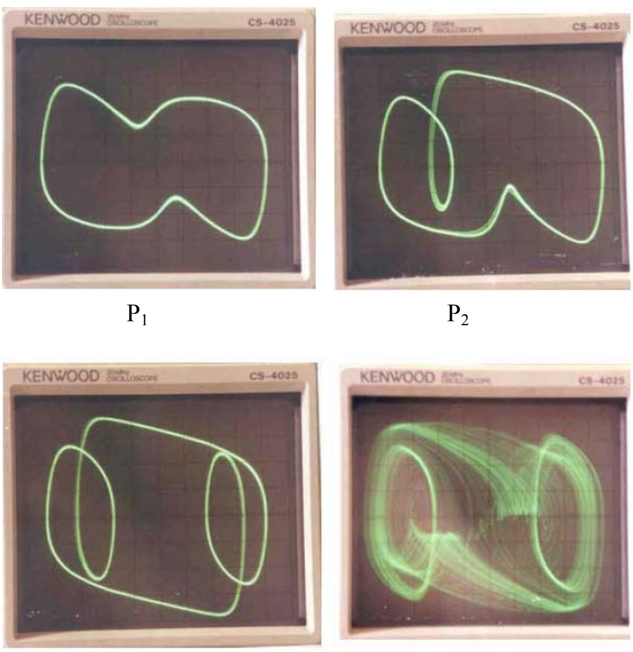
$$k_{01} = \frac{10^{-8} a_1}{R_1 R_7 C_1 C_2}\ ; \qquad k_{02} = \frac{10^{-8} a_2}{R_1 R_8 C_1 C_2}\ ;$$

The time unit is 10^{-4} S and $R_6 = 10^6 R_1^2 R_5 C_2^2$. A complete derivation of the differential equation (1) is accomplished by expressing the outputs voltages of the operational amplifiers and multipliers (see reference [3] derivation technique).

In order to control each parameter of equation (1) by varying only one resistor, we set the following values: R_1=9990Ω, R_2=1002Ω, R_5=9970Ω, R_6=99.7Ω, C_1=10.01nF, C_2=10.01nF and $a_1 = \frac{1}{\sqrt{2}} V_{eff}$. Thus, the coefficients ω, ε_{01}, k_{01} and k_{02} will respectively be controlled by R_4, R_3, R_7 and R_8. It is important to mention that the analog voltages obtained from our simulator are directly equivalent to the dimensionless variable x of equation (1).

The frequency $F_1 = \frac{\omega_1}{2\pi}$ of the excitation is monitored in order to study its effect on the bifurcations of the attractor x.

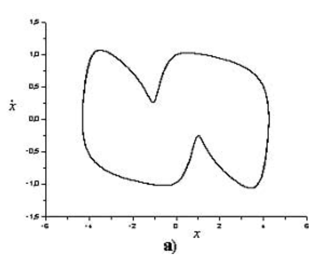
We set $R_3 = 1390\Omega$, $R_7 = 3000\Omega$, $R_4 = 9990\Omega$ and . $R_8 = \infty$. F_1 is monitored in the following window $0.0275\text{Hz} \le F_1 \le 0.0410\text{Hz}$. The pictures ($P_1$, P_2, P_3 and P_4) of Fig. 3. are the experimental phase portraits of the attractor x obtained respectively for F_1= *0.0410 Hz*, F_1= *0.0385 Hz,* F_1= *0.0360 Hz,* and F_1= *0.0275 Hz*. The system follows the following bifurcations as F_1 decreases: period 1-bifurcation $(P_1) \rightarrow$ period 2-bifurcation $(P_2) \rightarrow$ period 3-bifurcation $(P_3) \rightarrow$ chaotic bifurcation (P_4). This sequence of bifurcations shows period- adding transition route to chaos.



P_1 P_2 P_3 P_4

Figure 3. Experimental phase portraits of the attractor x (the corresponding parameters are defined in the text).

We have carried out a direct numerical simulation of the model described by Eq. (1). Fig. 4 shows the numerical phase portraits of the attractor x obtained for the same sets of parameters in Fig. 3.



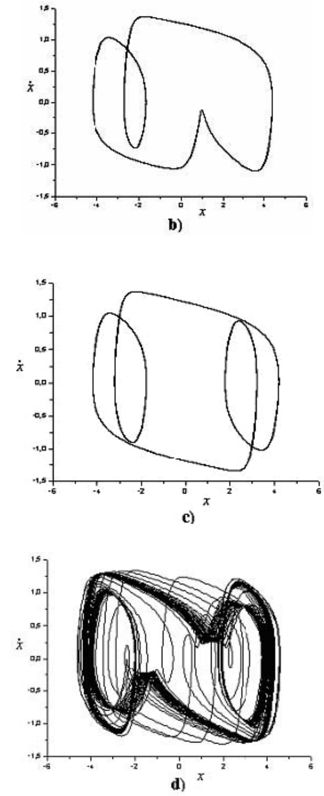


Figure 4. Numerical phase portraits of the attractor x (The corresponding parameters are defined in the text).

The results from the electronic simulator were very close to numerical ones. Table-I shows a comparison between some numerical and experimental bifurcation values of the control parameters F_1. A good agreement is obtained between the experimental phase portraits and the numerical ones. The experimental investigations reveal the extreme sensitivity of the system to small changes in F_1. Small windows of chaotic behaviour appear separated by domains of regular motion. The experimental study confirmed the existence of complex bifurcations such as torus breakdown transition route, period-adding scenario, period-doubling and sudden transition phenomena to chaos.

Table 1. Comparison of the bifurcation values for both numerical and experimental computations.

Control	Transitions ($m \rightarrow n$)		Bifurcation Values		
Parameters	From	To	Numerical	Experimental	Figures
	Period m	Period n	Values	Values	
F_1	1	2	0.0406	0.0409*Hz*	Figs 3 and 4
	2	3	0.0378	0.0380*Hz*	
	3	Chaos	0.0302	0.0308*Hz*	

V. CONCLUSION

The evaluation of analog simulation of the dynamics of nonlinear systems is proposed. The advantages and limits of both analog and numerical simulations are discussed. We have proposed some practical advices to tackle some difficulties encountered during the realization of electronic simulations. A model (Equation) describing the dynamics of a Rayleigh oscillator subjected to an external quasi-periodic excitation was considered to illustrate the concepts. We have proposed a real electronic prototype for the analysis of the Rayleigh system. The results from the analog simulator were compared with the numerical results and we found a very good agreement. One of the aims of this work was to encourage scientists to deal with analog simulations technique. We have shown that this simulation is suitable than its numerical counterpart for the analysis of nonlinear phenomena.

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